



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

October/November 2023

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

1 (a) By considering $(r+1)^2 - r^2$, use the method of differences to prove that

$$\sum_{r=1}^n r = \frac{1}{2}n(n+1). \quad [4]$$

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Given that $\sum_{r=1}^n (r+a) = n$, find a in terms of n . [3]

[illegible]

2 Prove by mathematical induction that, for all positive integers n ,

$$1 + 2x + 3x^2 + \dots + nx^{n-1} = \frac{1 - (n+1)x^n + nx^{n+1}}{(1-x)^2}. \quad [6]$$

[illegible]

3 The quartic equation $x^4 + bx^3 + cx^2 + dx - 2 = 0$ has roots $\alpha, \beta, \gamma, \delta$. It is given that

$$\alpha + \beta + \gamma + \delta = 3, \quad \alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 5, \quad \alpha^{-1} + \beta^{-1} + \gamma^{-1} + \delta^{-1} = 6.$$

(a) Find the values of b , c and d . [6]

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(b) Given also that $\alpha^3 + \beta^3 + \gamma^3 + \delta^3 = -27$, find the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [2]

[illegible]

4 The lines l_1 and l_2 have equations

$$\mathbf{r} = -2\mathbf{i} - 3\mathbf{j} - 5\mathbf{k} + \lambda(-4\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$$

respectively.

(a) Find the shortest distance between l_1 and l_2 . [5]

[illegible]

The plane Π contains l_1 and the point with position vector $-\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$.

- (b)** Find an equation of Π , giving your answer in the form $ax + by + cz = d$. [4]

[illegible]

5 Let k be a constant. The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are given by

$$\mathbf{A} = \begin{pmatrix} 1 & k & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 5 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 & -2 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{C} = \begin{pmatrix} -2 & -1 & 1 \\ 1 & 1 & 3 \end{pmatrix}.$$

It is given that $\mathbf{A}$ is singular.

(a) Show that $\mathbf{CAB} = \begin{pmatrix} 3 & -7 \\ -9 & 3 \end{pmatrix}$. [5]

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(b) Find the equations of the invariant lines, through the origin, of the transformation in the $x-y$ plane represented by **CAB**. [5]

[illegible]

[illegible]

(c) The matrices **D**, **E** and **F** represent geometrical transformations in the x - y plane.

- **D** represents an enlargement, centre the origin.
- **E** represents a stretch parallel to the x -axis.
- **F** represents a reflection in the line $y = x$.

Given that $\mathbf{CAB} = \mathbf{D} - 9\mathbf{EF}$, find $\mathbf{D}$, $\mathbf{E}$ and $\mathbf{F}$.

[5]

[illegible]

- 6 (a) Show that the curve with Cartesian equation

$$\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4}$$

has polar equation $r = \cos \theta$. [3]

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The curves C_1 and C_2 have polar equations

$$r = \cos \theta \quad \text{and} \quad r = \sin 2\theta$$

respectively, where $0 \leq \theta \leq \frac{1}{2}\pi$. The curves C_1 and C_2 intersect at the pole and at another point P .

- (b) Find the polar coordinates of P . [3]

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- (c) In a single diagram sketch C_1 and C_2 , clearly identifying each curve, and mark the point P . [3]

- (d)** The region R is enclosed by C_1 and C_2 and includes the line OP .

Find, in exact form, the area of R .

[6]

[illegible]

- 7 The curve C has equation $y = f(x)$, where $f(x) = \frac{x^2 + 2}{x^2 - x - 2}$.

(a) Find the equations of the asymptotes of C .

[2]

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(b) Find the coordinates of any stationary points on C , giving your answers correct to 1 decimal place.

[4]

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- (c) Sketch C , stating the coordinates of any intersections with the axes. [3]

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- (d) Sketch the curve with equation $y = \frac{1}{f(x)}$. [2]

(e) Find the set of values for which $\frac{1}{f(x)} < f(x)$. [4]

[illegible]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

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